Name: ……………………………… **M9L3 Circular and Periodic Functions** date: ………………………

Learn: unit circle

Key Concept • Functions on a Unit Circle

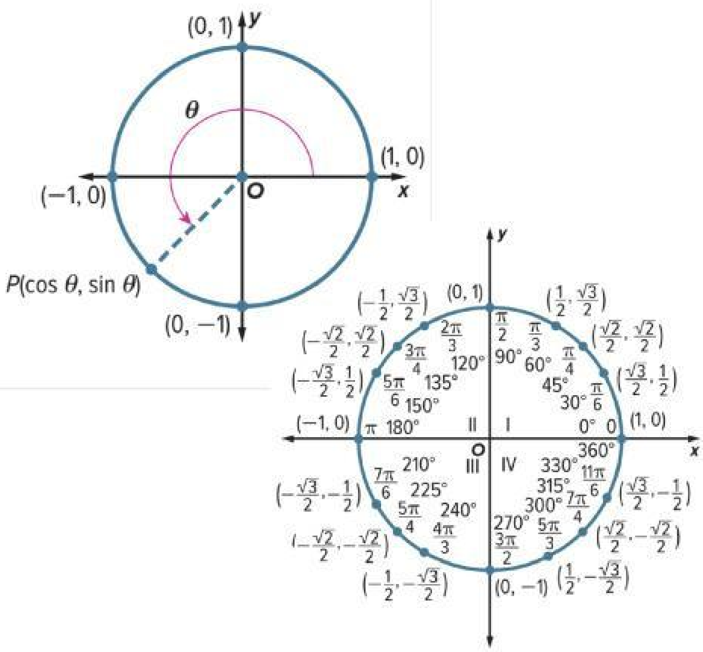
Words: If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, then $\cos\theta = x$ and $\sin\theta = y$.

Symbols: $P(x, y) = P(\cos\theta, \sin\theta)$

Example:

If $\theta = \frac{5\pi}{4}$,

$P(x, y) = P\left(\cos\frac{5\pi}{4}, \sin\frac{5\pi}{4}\right)$.



Learn: circular function

Both $\cos\theta = x$ and $\sin\theta = y$ are functions of θ. Because they are defined using a unit circle, they are **circular functions**, which describe a point on a circle as the function of an angle defined in radians.

Select only the circular functions (all that apply):

- $sin\theta$
- $\cos\theta$
- $\tan\theta$
- $\csc\theta$
- $\sec\theta$
- $\cot\theta$

solve: find sin and cos given a point (P) on the circle

The terminal side of θ in standard position intersects the unit circle at $P\left(-\frac{12}{13}, \frac{5}{13}\right)$.

Find cos θ and sin θ.

$\cos\theta =$ ……… $\sin\theta =$ ………

The terminal side of θ in standard position intersects the unit circle at $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$. Find cos θ and sin θ. Write the solutions as decimals.

$\cos\theta =$ ……… $\sin\theta =$ ………

Hint: $\cos\theta = x, \sin\theta = y$

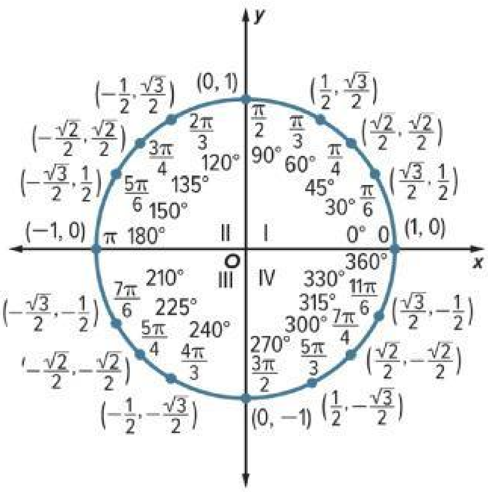
solve: find sin and cos given a point (P) on the circle

Find the exact values of the six trigonometric functions for an angle that measures $\frac{5\pi}{4}$ radians. Use the unit circle

$\sin\theta =$ ……… $\csc\theta =$ ………

$\cos\theta =$ ……… $\sec\theta =$ ………

$\tan\theta =$ ……… $\cot\theta =$ ………



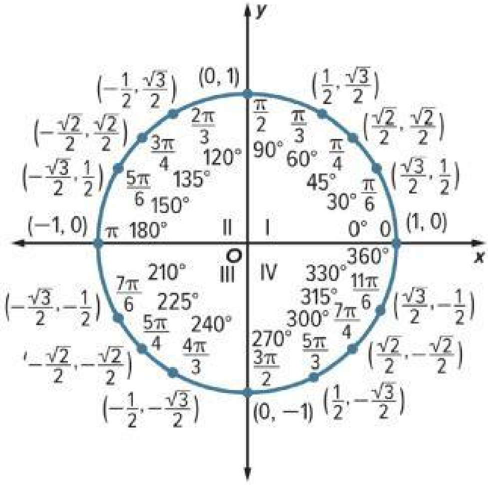
Hint: $\cos\theta = x, \sin\theta = y$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

solve: find sin and cos given a point (P) on the circle

Find the exact values of the six trigonometric functions for an angle that measures $\frac{4\pi}{3}$ radians. Use the unit circle



$\sin\theta =$ ... ... $\csc\theta =$ ... ...

$\cos\theta =$ ... ... $\sec\theta =$ ... ...

$\tan\theta =$ ... ... $\cot\theta =$ ... ...

Hint: $\cos\theta = x, \sin\theta = y$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Learn: periodic function

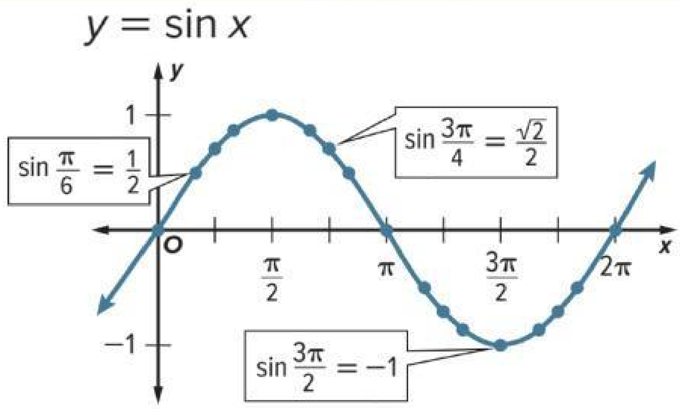
A **periodic function** has *y*-values that repeat at regular intervals. One complete pattern of a periodic function is called a **cycle,** and the horizontal length of one cycle is called the **period**.

The values of the sine and cosine functions can be found by using the unit circle. As you move around the unit circle, the values of these functions repeat every 360° or 2π. So, the sine and cosine functions are periodic functions where $\sin(x + 2\pi) = \sin x$ and $\cos(x + 2\pi) = \cos x$. Because tangent, cosecant, secant, and cotangent can be defined in terms of sine and cosine, they are also periodic functions.

Select only the **periodic functions** (all that apply):

- $sin\theta$
- $\cos\theta$
- $\tan\theta$
- $\csc\theta$
- $\sec\theta$
- $\cot\theta$

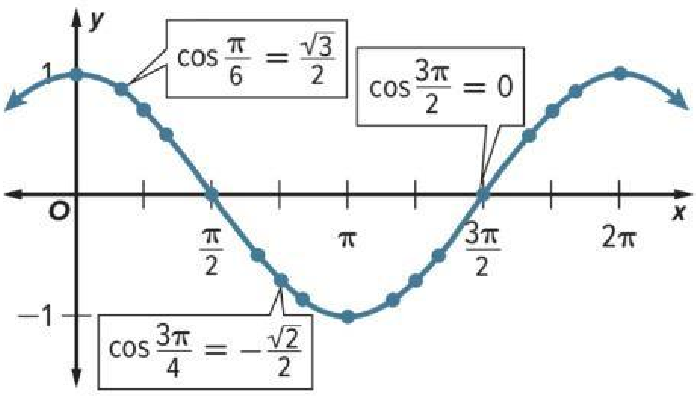
Learn: graph y = sinθ



What is the **period** of the function sinθ (when it finished a full rotation):

- $\frac{\pi}{2}$
- π
- 2π

Learn: graph y = cosθ



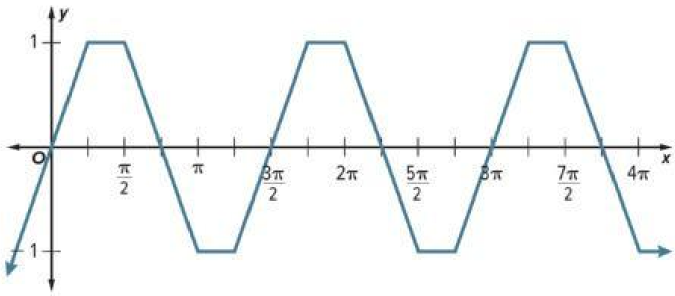
What is the **period** of the function cosθ (when it finished a full rotation):

- $\frac{\pi}{2}$
- π
- 2π

Solve: identify the period of the function

Example 3 Identify the Period of a Function

Determine the period of the function.



What is the **period** of the function $\cos\theta$ (when it finished a full rotation):

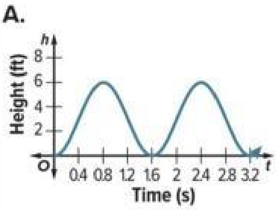
- $\frac{\pi}{2}$
- π
- $\frac{3\pi}{2}$

Solve: identify the period of the function

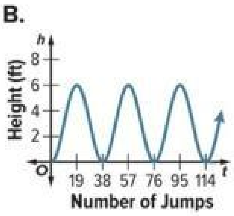
Check

POGO STICK In 2016, Henry Cabelus set the record for the fewest pogo stick jumps in one minute, when he jumped up and down only 38 times in one minute. Cabelus's height off the ground *h* while jumping is a function of time *t*. Suppose that at the highest point of each jump, Cabelus was 6 feet off the ground. Select the graph of the function.

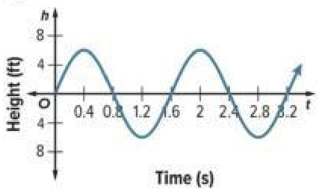
A.



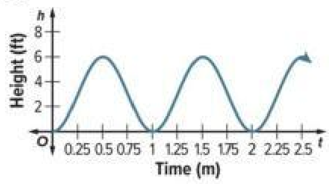
B.



C.



D.



Example 5 Evaluate Trigonometric Expressions

Find the exact value of $\cos \frac{10\pi}{3}$.

$\cos \frac{10\pi}{3} = \cos\left(\frac{4\pi}{3} + \frac{6\pi}{3}\right)$ $\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{10\pi}{3}$

$= \cos \frac{4\pi}{3}$ $\cos(x + 2\pi) = \cos x$

$= -\frac{1}{2}$ Use the unit circle.

Check

Find the exact value of each expression.

$\sin \frac{8\pi}{3} =$ ______

$\cos \frac{5\pi}{6} =$ ______

$\sin \frac{21\pi}{4} =$ ______

$\cos \frac{11\pi}{3} =$ ______

$\cos 60° =$ ______

$\sin 600° =$ ______