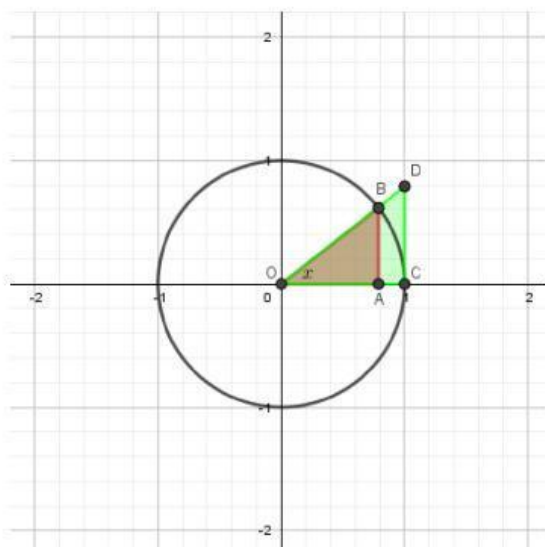


RUMUS DASAR LIMIT FUNGSI TRIGONOMETRI

Perhatikan gambar berikut!



Pada gambar di samping, diketahui,

Panjang $OB = OC = 1$ (jari-jari lingkaran) dan besar $\angle AOB = x$,

Maka:

$$1. \sin x = \frac{AB}{OB} \leftrightarrow AB = OB \times \sin x$$

$$AB = \sin x$$

$$2. \cos x = \frac{OA}{OB} \leftrightarrow OA = OB \times \cos x$$

$$OA = \cos x$$

$$3. \tan x = \frac{CD}{OC} \leftrightarrow CD = OC \times \tan x$$

$$CD = \tan x$$

Luas $\triangle AOB$, Luas juring COB , dan Luas $\triangle COD$ dapat dituliskan sebagai berikut:

$$1. \text{Luas } \triangle AOB = \frac{1}{2} \times OA \times AB$$

$$\text{Luas } \triangle AOB = \frac{1}{2} \times \cos x \times \sin x$$

$$2. \text{Luas juring } COB = \frac{x}{2\pi} \times \pi \times OB^2$$

$$\text{Luas juring } COB = \frac{1}{2} x$$

$$3. \text{Luas } \triangle COD = \frac{1}{2} \times OC \times CD$$

$$\text{Luas } \triangle COD = \frac{1}{2} \tan x$$

Hubungan Luas $\triangle AOB$, Luas Juring COB, dan Luas $\triangle COD$

$$L. \triangle AOB < L. \text{ juring } COB < L. \triangle COD$$

$$\frac{1}{2} \cdot \cos x \cdot \sin x < \frac{1}{2}x < \frac{1}{2}\tan x$$

$$\cos x \cdot \sin x < x < \tan x$$

$$\cos x \cdot \sin x < x < \frac{\sin x}{\cos x}$$

$$\frac{\cos x \cdot \sin x}{\sin x} < \frac{x}{\sin x} < \frac{\sin x}{\cos x \cdot \sin x}$$

$$\cos x < \frac{x}{\sin x} < \frac{1}{\cos x}$$

$$\lim_{x \rightarrow 0} \cos x < \lim_{x \rightarrow 0} \frac{x}{\sin x} < \lim_{x \rightarrow 0} \frac{1}{\cos x}$$

$$\cos 0 < \lim_{x \rightarrow 0} \frac{x}{\sin x} < \frac{1}{\cos 0}$$

$$1 < \lim_{x \rightarrow 0} \frac{x}{\sin x} < \frac{1}{1}$$

$$1 < \lim_{x \rightarrow 0} \frac{x}{\sin x} < 1$$

dikarenakan

$$1 < \lim_{x \rightarrow 0} \frac{x}{\sin x} < 1$$

maka

$$\text{Nilai } \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

Dengan mengembangkan pola perolehan nilai

$$\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

Maka diperoleh rumus dasar limit fungsi trigonometri:

$$1) \lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \frac{a}{b}$$

$$2) \lim_{x \rightarrow 0} \frac{ax}{\sin bx} = \frac{a}{b}$$

$$3) \lim_{x \rightarrow 0} \frac{\tan ax}{bx} = \frac{a}{b}$$

$$4) \lim_{x \rightarrow 0} \frac{ax}{\tan bx} = \frac{a}{b}$$

$$5) \lim_{x \rightarrow 0} \frac{\tan ax}{\tan bx} = \frac{a}{b}$$

$$6) \lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$$

$$7) \lim_{x \rightarrow 0} \frac{\tan ax}{\sin bx} = \frac{a}{b}$$

$$8) \lim_{x \rightarrow 0} \frac{\sin ax}{\tan bx} = \frac{a}{b}$$

Tentukan nilai limit berikut!

$$1. \lim_{x \rightarrow 0} \frac{3x}{\tan 6x} = \frac{3 \cdot (0)}{\tan 6 (0)} = \frac{0}{0} \text{ (bentuk tak tentu)}$$

Penggunaan rumus dasar:

$$\lim_{x \rightarrow 0} \frac{3x}{\tan 6x} = \frac{3}{6} = \frac{1}{2}$$

$$2. \lim_{x \rightarrow 0} \frac{\sin^3 2x}{x^3} = \frac{\sin^3 2(0)}{0^3} = \frac{0}{0} \text{ (bentuk tak tentu)}$$

Penggunaan rumus dasar:

$$\lim_{x \rightarrow 0} \frac{\sin^3 2x}{x^3} = \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{x} \right)^3 = \left(\frac{2}{1} \right)^3 = 8$$

$$3. \lim_{x \rightarrow 0} \frac{x \tan 3x}{\sin^2 2x} = \frac{0 \cdot \tan 3(\quad)}{\sin^2 2(0)} = -$$

Penggunaan rumus dasar:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x \tan 3x}{\sin^2 2x} &= \lim_{x \rightarrow 0} \frac{x}{\sin 2x} \times \lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 2x} \\ &= - \times - \\ &= - \end{aligned}$$

$$4. \lim_{x \rightarrow \frac{1}{2}} \frac{\sin(4x - 2)}{\tan(2x - 1)} = \lim_{x \rightarrow \frac{1}{2}} \frac{\sin(4(\quad) - 2)}{\tan(2(\quad) - 1)} = -$$

Penggunaan rumus dasar:

$$\lim_{x \rightarrow \frac{1}{2}} \frac{\sin(4x - 2)}{\tan(2x - 1)} = \lim_{x \rightarrow \frac{1}{2}} \frac{\sin 2(\quad x - \quad)}{\tan(2x - 1)} = - =$$

$$5. \lim_{x \rightarrow 1} \frac{\sin(x - 1)}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{\sin(\quad - 1)}{\quad^2 - 1} = -$$

Penggunaan rumus dasar:

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\sin(x - 1)}{x^2 - 1} &= \lim_{x \rightarrow 1} \frac{\sin(x - 1)}{(\quad + \quad)(\quad - \quad)} = \\ &= \lim_{x \rightarrow 1} \frac{\sin(x - 1)}{x - 1} \cdot \lim_{x \rightarrow 1} \frac{1}{\quad} \\ &= 1 \cdot \frac{\quad}{+} \\ &= - \end{aligned}$$