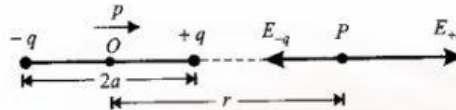


Unit-1 Electrostatics
(Electric Charges and Fields)

1. Derive the expression for electric field intensity at a point the axial line of a dipole.

As shown in the figure, consider an electric dipole of charges $-q$ and $+q$, separated by distance $2a$ and placed in vacuum. Let P be point on the axial line at a distance r from the centre O of the dipole on the side of the charge $+q$.



Electric field due to charge $-q$ at a point P is

$$\vec{E}_{-q} = \frac{-q}{4\pi\epsilon_0(\quad)^2} \hat{p}$$

Where $\hat{p}$ is a unit vector along the dipole axis from $-q$ to $+q$.

Electric field due to charge $+q$ at a point P is

$$\vec{E}_{+q} = \frac{q}{4\pi\epsilon_0(\quad)^2} \hat{p}$$

Hence the resultant electric field at point P is

$$\begin{aligned} \vec{E}_{axial} &= \vec{E}_{-q} + \vec{E}_{+q} \\ &= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(\quad)^2} - \frac{1}{(\quad)^2} \right] \hat{p} = \frac{q}{4\pi\epsilon_0} \cdot \frac{\quad}{(\quad^2 - \quad^2)^2} \hat{p} \end{aligned}$$

$$\boxed{\vec{E}_{axial} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\quad}{(\quad^2 - \quad^2)^2} \hat{p}}$$

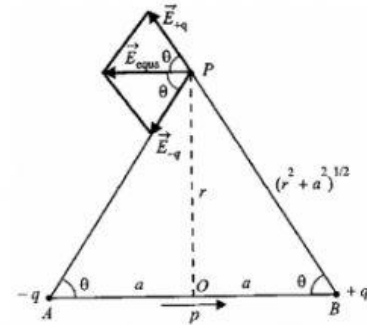
Here $p = q \times 2a$ = dipole moment. For $r \gg a$, a^2 can be neglected compared to r^2 .

$$\therefore \boxed{\vec{E}_{axial} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\quad}{\quad^3} \hat{p}}$$

Electric field at any point on the axis of the dipole acts along the dipole axis from _____ to _____ charge, i.e. in the direction of _____.

2. Derive the expression for the electric field intensity at any point along the equatorial line of an electric dipole.

As shown in the figure, consider an electric dipole of charges $-q$ and $+q$, separated by distance $2a$ and placed in vacuum. Let P be point on the equatorial line of the dipole at distance r from it, i.e. $OP = r$



Electric field at point P due to $+q$ charge is

$$\vec{E}_{+q} = \frac{q}{4\pi\epsilon_0(\underline{\quad}^2 + \underline{\quad}^2)} \text{ directed along BP.}$$

Electric field at point P due to $-q$ charge is

$$\vec{E}_{-q} = \frac{q}{4\pi\epsilon_0(\underline{\quad}^2 + \underline{\quad}^2)} \text{ directed along AP.}$$

Clearly, the magnitudes of $\vec{E}_{+q}$ and $\vec{E}_{-q}$ are equal. So, the components of $\vec{E}_{+q}$ and $\vec{E}_{-q}$ normal to the dipole axis will cancel out. The components parallel to the dipole axis will add up. The total electric field $\vec{E}_{eq}$ is opposite to $\vec{p}$.

$$\therefore \vec{E}_{eq} = -(E_{-q} \cos \theta + E_{+q} \cos \theta) \hat{p}$$

$$\vec{E}_{eq} = -\underline{\quad} \vec{E}_{+q} \cos \theta \hat{p} \quad (\because E_{+q} = E_{-q})$$

$$\text{or, } \vec{E}_{eq} = -2 \frac{q}{4\pi\epsilon_0(\underline{\quad}^2 + \underline{\quad}^2)} \cos \theta \hat{p}$$

Now, from fig. $\cos \theta = \frac{a}{\sqrt{r^2 + a^2}}$,

$$\therefore \vec{E}_{eq} = -2 \frac{q}{4\pi\epsilon_0(\underline{\quad}^2 + \underline{\quad}^2)} \frac{a}{\sqrt{r^2 + a^2}} \hat{p}$$

$$\text{or, } \boxed{\vec{E}_{eq} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{\underline{\quad}}{(\underline{\quad}^2 + \underline{\quad}^2)^{3/2}} \hat{p}} \quad \text{where } \underline{\quad} = 2qa, \text{ is electric dipole moment}$$

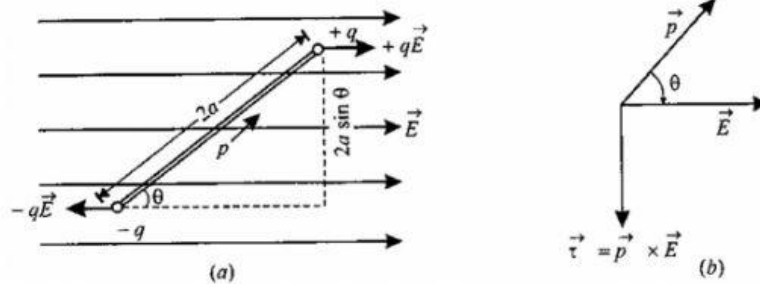
If the point P is located far away from the dipole, $r \gg a$, then

$$\boxed{\vec{E}_{eq} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{\underline{\quad}}{\underline{\quad}^3} \hat{p}}$$

The direction of electric field at any point on the equatorial line of the dipole will be antiparallel to the dipole moment $\hat{p}$.

The electric field intensity due to a short dipole at a distance ' r ' along its axis is $\underline{\quad}$ the intensity at the same distance along the equatorial axis.

3. Derive the expression for the torque acting on an electric dipole, when held in a uniform electric field. (net translational force is zero)



Consider an electric dipole consisting of charges $+q$ and $-q$ and length $2a$ placed in a uniform electric field $\vec{E}$, making an angle θ with it. It has a dipole moment of magnitude,

$$p = \underline{\hspace{2cm}}$$

Force exerted on charge $+q$ by field $\vec{E} = +\underline{\hspace{1cm}}\vec{E}$ along $\vec{E}$

Force exerted on charge $-q$ by field $\vec{E} = -\underline{\hspace{1cm}}\vec{E}$ opposite to $\vec{E}$

$$\therefore \vec{F}_{total} = +\underline{\hspace{1cm}}\vec{E} - \underline{\hspace{1cm}}\vec{E} = \underline{\hspace{2cm}}$$

Hence net translating force on dipole in a uniform electric field is $\underline{\hspace{2cm}}$. But the two equal and opposite force act at different points on the dipole. So, they form a couple and exert $\underline{\hspace{2cm}}$.

$\underline{\hspace{2cm}}$ = either force $\times$ perpendicular distance between the two forces

$$i.e. \tau = \underline{\hspace{1cm}}E \times 2a \underline{\hspace{1cm}} \theta = (\underline{\hspace{1cm}} \times 2a)E \underline{\hspace{1cm}} \theta$$

$$\boxed{\tau = \underline{\hspace{1cm}}E \underline{\hspace{1cm}} \theta}$$

As the direction of torque $\vec{\tau}$ is perpendicular to both $\vec{p}$ and $\vec{E}$, we can write

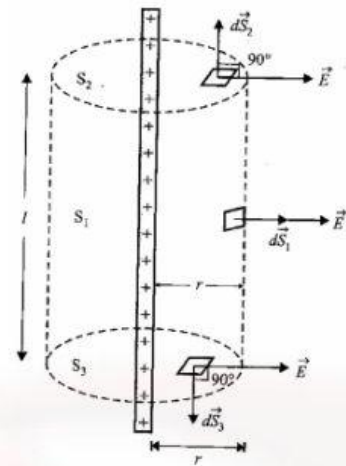
$$\boxed{\vec{\tau} = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}$$

As shown in the figure the direction of torque is that in which a right-handed screw would advance when rotated from $\underline{\hspace{1cm}}$ to $\underline{\hspace{1cm}}$.

4. Derive the expression for the electric field intensity due to infinitely long, straight wire of linear charge density $\lambda \text{ Cm}^{-1}$.

Consider a thin infinitely long straight wire having a uniform - _____ charge density $\lambda \text{ Cm}^{-1}$. By symmetry, the field $\vec{E}$ of the line charge is directed radially _____ and its magnitude is same at all points equidistant from the line charge. Here we choose a _____ Gaussian surface of radius r , length l and with its axis along the line charge.

As shown in the figure the Gaussian surface has curved surfaces S_1 and flat circular ends S_2 and S_3 .



Since $\vec{dS_1} \parallel \vec{E}$, $\vec{dS_2} \perp \vec{E}$ and $\vec{dS_3} \perp \vec{E}$ only the curved surface contributes towards the total flux.

$$\phi_E = \oint_S \vec{E} \cdot \vec{ds} = \int_{S_1} \vec{E} \cdot \vec{ds_1} + \int_{S_2} \vec{E} \cdot \vec{ds_2} + \int_{S_3} \vec{E} \cdot \vec{ds_3}$$

$$\phi_E = \int_{S_1} E dS_1 \cos 0^\circ + \int_{S_2} E dS_2 \cos 90^\circ + \int_{S_3} E dS_3 \cos 90^\circ$$

$$\phi_E = E \int_{S_1} dS_1 = E \times \text{area of curved surface}$$

$$\phi_E = E \times 2\pi \text{ ---}$$

Charged enclosed by the Gaussian surface is $q = \lambda \text{ ---}$

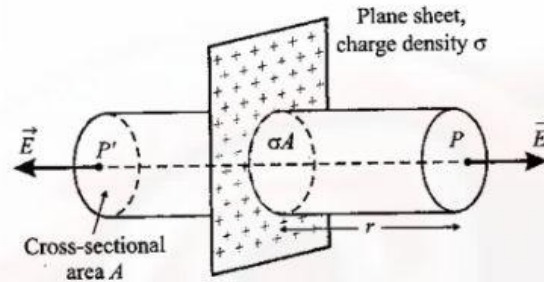
Using Gauss's theorem, $\phi_E = \frac{q}{\epsilon_0}$, we get

$$E \cdot 2\pi \text{ ---} = \frac{\lambda \text{ ---}}{\epsilon_0}$$

$$\text{or, } \boxed{E = \frac{\lambda}{2\pi\epsilon_0 \text{ ---}}}$$

5. Derive the expression for the electric field intensity at a point near a thin infinite plane sheet of charge density $\sigma \text{ Cm}^{-2}$.

As shown in the figure consider a thin, infinite plane sheet of charge with uniform _____ charge density σ . We have to calculate its electric field at a point P at distance r from it. By symmetry, electric field $\vec{E}$ points _____ normal to the sheet. Also, it must have same magnitude and opposite direction at two points P and P' equidistant from the sheet and on opposite sides.



Here we consider a _____ Gaussian surface of cross-sectional area A and length $2r$ with its axis perpendicular to the sheet. As the lines of force are parallel to the curved surface of the cylinder, the flux through the curved surface is _____. The flux through the plane end faces of the cylinder is

$$\phi_E = EA + EA = 2EA$$

Using Gauss's theorem, $\phi_E = \frac{\sigma A}{\epsilon_0}$, we get

$$\begin{aligned} \text{---} EA &= \frac{\sigma \text{---}}{\epsilon_0} \\ \text{or, } E &= \boxed{\frac{\sigma}{\text{---} \epsilon_0}} \end{aligned}$$

6. Calculate the electric field due to a uniformly charged spherical shell at a point (i) outside the shell, (ii) on the shell and (iii) inside the shell.

Consider a thin spherical shell of charge of radius R with uniform _____ charge density σ . From symmetry, we see that the electric field $\vec{E}$ at any point is _____ and has same magnitude at points equidistant from the centre. To determine electric field at any point at a distance r from O , we choose a concentric sphere of radius r as the _____ surface.

(i) When point P lies outside the spherical shell: The total charge q inside the Gaussian surface is the charge on the shell of radius R and area $4\pi R^2$.

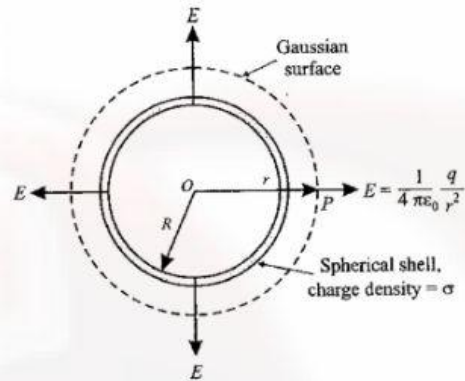
$$\therefore q = 4\pi R^2 \sigma$$

Flux through the Gaussian surface, $\Phi_E = E \times 4\pi r^2$

Using Gauss's theorem, $\Phi_E = \frac{q}{\epsilon_0}$, we get

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\text{or, } E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$



For points outside the shell, the field due to uniformly charged shell is as if the entire charge of shell is concentrated at its _____.

(ii) When point P lies on the spherical shell: The Gaussian surface just encloses the charged spherical shell. Applying Gauss's theorem, for $r=R$

$$E \times 4\pi R^2 = \frac{q}{\epsilon_0}$$

$$\text{or, } E = \frac{q}{4\pi\epsilon_0 R^2}$$

$$\text{or, } \boxed{E = \frac{\sigma}{\epsilon_0}} \quad \text{since } q = 4\pi R^2 \sigma$$

(iii) When point P lies inside the spherical shell: For $r < R$, the charge enclosed by Gaussian surface is _____

Flux through the Gaussian surface, $\Phi_E = E \times 4\pi r^2$

Using Gauss's theorem, $\Phi_E = \frac{q}{\epsilon_0}$, we get

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0} \quad \text{or } E = \frac{q}{4\pi\epsilon_0 r^2}$$

Hence, electric field due to a uniformly charged spherical shell is _____ at all points inside the shell.