2021年第十七届“IMC國際數學交流活動”（新加坡）

Seventeenth IMC International Mathematics Contest (Singapore), 2021

Grade 6 Grand Finals Contest Problems

Time Limit: 90 seconds **Total Scores: 100 points**

Printed Name: _____ Country: _____ Score: _____

Part I. Multiple Choice Problems. (Each problem is worth 5 points, for a total of 40 points)

Chọn câu trả lời đúng:

1. What is the simplified value of $\frac{2021}{2022}+\frac{2021+20212021}{2022+20222022}$?

A. 2021 B. 1 C. $\frac{2021}{2022}$ D. $\frac{2021}{1011}$

2. There are two numbers: A and B. If 36% of A is only $\frac{1}{3}$ of the 1.5 times of B, the value of B is how many times the value of A?

A. 0.68 B. 1.36 C. 2.72 D. 3.2

3. A road construction team built an average of 117 meters of road every day in the first 10 days. After adding some workers, the roads had been constructed at an average of 180 meters per day in the next 8 days. How many meters of roads were built averagely every day in 18 days?

A. 120 B. 145 C. 148.5 D. 152

4. If the sum of several positive integers is 50, then what is the largest possible product of these integers?

A. 625 B. 4624 C. 500000 D. $3^{16}\times 2$

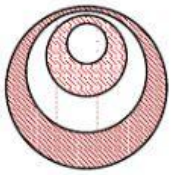
5. When a nonzero positive integer is divided by 8, the remainder is 1. When this number is divided by 9, the remainder is 7. When the same number is divided by 36, what is the remainder?

A. 1 B. 7 C. 25 D. 35

6. A nonzero positive integers has four divisors, and the sum of two of its prime divisors is 90. What is the largest possible value of that integer?

A. 1961 B. 2021 C. 2025 D. 2173

7. As shown in the figure, the ratio of radius of the four circles is 1:2:3:4, and the total area of the unshaded regions is 30 square units. What is the total area of the shaded regions?



A. 30 B. 31.4 C. 50 D. 62.8

8. There are four cards. Each card contains the digits (0 and 1); (2 and 3); (4 and 5); (6 and 7) on either the front or back parts. If the digits on the four cards are arranged as a four-digit number, (when the digit in the front part is used, then the digit on the other part can't be used to form the four-digit number), then how many such four-digit numbers are there?

A. 48 B. 336 C. 384 D. 438

LIVEWORKSHEETS

Part II. Short Answer Questions (Each problem is worth 5 points for a total of 40 points)

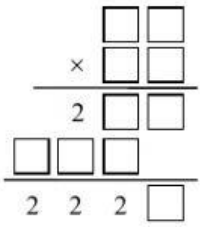
Điền đáp số, không cần giải thích:

9. Find the simplified value of $1 \times 1 + 3 \times 3 + 5 \times 6 + 7 \times 10 + 9 \times 15 + \ldots + 19 \times 55$.

10. What is the simplified value of $\frac{1}{1\times5}+\frac{1}{5\times9}+\frac{1}{9\times13}+\frac{1}{13\times17}+\cdots+\frac{1}{2017\times2021}$?

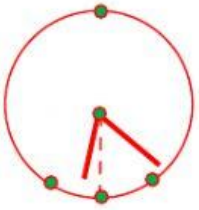
11. Arrange two different two-digit numbers side by side forming a four-digit number such that this four-digit number is 29 less than 50 times the sum of those two-digit numbers. What is the sum of all the possible four digit numbers that satisfy the condition?

12. Fill in an odd number in each of the boxes of the multiplication sentence to make the expression true. What is the sum of the two factors?

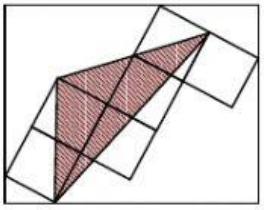


13. A pharmaceutical laboratory produces a normal saline solution with a 0.9% salt content. At present, there are 150kg saline solution with 6% salt content and 450kg saline solution with 3.2% salt content. The two kinds of saline solutions are used up, and then pure water is used as normal saline. How much kg of pure water is it necessary to add?

14. Between 6 o'clock and 7 o'clock, the hour hand and the minute hand are located on both sides of "6" on the clock face. Connect the center of the clock face with "6", and the angle between it and the minute hand is equal to 3 times of the angle between it and the hour hand. How many minutes are there past 6 o'clock?



15. As shown in the figure, six squares of the same size are placed in the rectangle with the length of 77 cm and the width of 60 cm. There are four contact points on the boundary between the small squares and the large rectangle. What is the area of the grid-shaded triangle in the figure in square cm?



16. To maintain physical distance during the COVID19 pandemic, the school prohibits persons to sit next to each other. Therefore, the canteen management has changed the seat capacity policy from the usual 12 persons in a row to 5 persons in a row at the dining table. How many ways can the usual 12 people long table be arranged in a row with 5 seats?

Part III. Problem Solving. (Each problem is worth 10 points for a total of 20 points. Simplified Solution of each problem is a must and it worth 4 points)

Trình bày lời giải chi tiết:

17. Processing a batch of parts, group A and group B should work together for 6 hours. If group A processes 8 more parts per hour, when the two groups complete the job together, group A makes 20 more parts than the original plan; if group B processes 8 less parts per hour, when the two groups finish the job together, group A processes 30 more parts than the original plan. Then how many part pieces this batch has processed?

18. How many whole numbers from 1 to 120 cannot be expressed as the sum of the squares of two whole numbers?

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