Nama : .....

Kelas/No. : .....

MATEMATIKA PEMINATAN

KELAS XII

Kompetensi Dasar : 3.3 Menggunakan prinsip turunan ke fungsi trigonometri

4.3 Menyelesaikan masalah yang berkaitan dengan turunan fungsi trigonometri

A. Definisi Turunan

Turunan fungsi $f(x)$ didefinisikan sebagai:

$$f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

B. Dasar Turunan Fungsi Aljabar

1. $f(x) = x^n$, maka $f'(x) =$ ... x ...
2. $f(x) = ax^n$, maka $f'(x) =$ ... ... x ...

C. Sifat-sifat Turunan Fungsi Aljabar

Jika diketahui k suatu konstanta, $u = u(x), v = v(x)$, dan masing-masing mempunyai turunan $u'(x)$ dan $v'(x)$, maka berlaku:

1. $f(x) = u + v$, maka $f'(x) = u' + v'$
2. $f(x) = u - v$, maka $f'(x) =$ ... $-$ ...
3. $f(x) = uv$, maka $f'(x) = u'.$ ... $+$ ... .
4. $f(x) = f(u)$, maka $f'(x) = f'(u).u'$
5. $f(x) = \frac{u}{v}$, maka $f'(x) = \frac{\ldots . \ldots - \ldots . \ldots}{\ldots \ldots}$

D. Mengingat Kembali Turunan pada Fungsi Aljabar

1. $f(x) = 5$

   $f'(x) =$ ...

2. $f(x) = x$

   $f'(x) =$ ...

3. $f(x) = -3x^2$

   $f'(x) =$ ...

4. $f(x) = (3x)^2$

   $f'(x) =$ ... (...) ...

   $f'(x) =$ ...

5. $f(x) = 5(3x^2 - 7x)^4$

$f'(x) =$ ... ... (... ··· − ...) ··· (... − ...)

$f'(x) =$ ... (... − ...)(... ··· − ...) ···

E. Turunan Fungsi Trigonometri

1. Turunan dari $y = f(x) = \sin x$

Diketahui $f(x) = \sin x$ maka $f(x + h) = \sin($ ... + ...$)$

$$f'(x) = \frac{dy}{dx} = \frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\ldots(\ldots + \ldots) - \ldots}{\ldots}$$

$$= \lim_{h \to 0} \frac{\ldots \ldots \left(\frac{\ldots + \ldots + \ldots}{2}\right) \sin\left(\frac{\ldots + \ldots - \ldots}{2}\right)}{\ldots}$$

$$= \ldots \lim_{h \to 0} \frac{\ldots \left(\frac{\ldots + \ldots}{2}\right) \sin\left(\frac{\ldots}{2}\right)}{\ldots}$$

$$= \ldots \lim_{h \to 0} \frac{\ldots \left(\frac{\ldots + \ldots}{2}\right) \sin\left(\frac{1}{2}h\right)}{\ldots}$$

$$= \ldots \lim_{h \to 0} \ldots \left(\frac{\ldots + \ldots}{2}\right) \lim_{h \to 0} \frac{\ldots(\ldots)}{\ldots}$$

$$= \ldots . \ldots \left(\frac{\ldots + \ldots}{2}\right) \frac{\ldots / \ldots}{\ldots}$$

$$= \ldots . \cos \frac{\ldots}{2} . \frac{\ldots}{\ldots}$$

$$= \ldots . \frac{\ldots}{\ldots} . \cos \ldots$$

$$= \ldots$$

Jadi turunan dari $f(x) = \sin x$ adalah $f'(x) = \cos x$

2. Turunan dari $y = f(x) = \cos x$

Diketahui $f(x) = \cos x$ maka $f(x+h) = \cos(\ \ldots\ +\ \ldots)$

$$f'(x) = \frac{dy}{dx} = \frac{df(x)}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\ldots\ (\ \ldots + \ldots) - \ \ldots}{\ldots}$$

$$= \lim_{h\to 0} \frac{\ldots\ \ldots \left(\frac{\ldots + \ldots + \ldots}{2}\right) \sin\left(\frac{\ldots + \ldots - \ldots}{2}\right)}{\ldots}$$

$$= \ldots \lim_{h\to 0} \frac{\ldots \left(\frac{\ldots + \ldots}{2}\right) \sin\left(\frac{\ldots}{2}\right)}{\ldots}$$

$$= \ldots \lim_{h\to 0} \frac{\ldots \left(\frac{\ldots + \ldots}{2}\right) \sin\left(\frac{1}{2}h\right)}{\ldots}$$

$$= \ldots \lim_{h\to 0} \ldots \left(\frac{\ldots + \ldots}{2}\right) \lim_{h\to 0} \frac{\ldots \left(- \ldots\right)}{\ldots}$$

$$= \ldots . \ldots \left(\frac{\ldots + \ldots}{2}\right) \frac{\ldots / \ldots}{\ldots}$$

$$= \ldots \ . \sin \frac{\ldots}{2} \ . \ \frac{\ldots}{\ldots}$$

$$= \ldots \ . \frac{\ldots}{\ldots} \ . \sin \ \ldots$$

$$= \ldots$$

Jadi turunan dari $f(x) = \cos x$ adalah $f'(x) = -\sin x$

3. $f(x) = x + \cos x$

$f'(x) = \ \ldots + (\ \ \ldots\ \ )$

$f'(x) = \ \ldots - \ \ \ldots.$

4. $f(x) = 3\sin x - 4\cos x$

$f'(x) = \ \ \ldots\ - \ldots\ (\ \ \ldots\ \ )$

$f'(x) = \ \ \ldots\ + \ \ldots$

5. $f(x) = x.\sin x$

$u = x, u' = \ldots$

$v = \ldots \quad , v' = \ldots$

$f'(x) = \ldots \ . \ \ldots + \ldots \ . \ \ldots$
$f'(x) = \ldots + \ldots \ \ldots$

6. $f(x) = x^2.\cos x$

$u = x^2, u' = \ldots$

$v = \ldots \quad , v' = \ldots$

$f'(x) = \ldots \ . \ \ldots + \ldots^2 \ .(\ \ldots \ )$
$f'(x) = \ldots \ \ldots - \ \ldots^2 \ \ldots$

7. $f(x) = \sin x \, . \cos x$

$u = \ldots \quad , u' = \ldots$

$v = \ldots \quad , v' = \ldots$

$f'(x) = \ldots \ . \ \ldots + \ldots \ .(\ \ldots \ )$

$f'(x) = \ldots \ \ldots \ \ldots - \ \ldots \ \ldots \ \ldots$

$f'(x) = \ldots$ (identitas trigonometri)

8. $f(x) = \frac{\sin x}{\cos x}$

$u = \ldots \quad , u' = \ldots$

$v = \ldots \quad , v' = \ldots$

$f'(x) = \frac{\ldots \ . \ \ldots - \ldots \ .(\ \ldots \ )}{(\ \ldots \ ) \ \ldots}$

$f'(x) = \frac{\ldots \ \ldots \ \ldots + \ldots \ \ldots \ \ldots}{\ldots \ \ldots \ \ldots}$

$f'(x) = \frac{\ldots}{\ldots \ \ldots \ \ldots}$

$f'(x) = \ldots \ \ldots \ \ldots$