Lembar Kerja Peserta Didik (LKPD)

Topik: Operasi Vektor di R3

Nama : ....

Kelas : ....

No Absen : ....

Operasi Vektor di R3

Jika dua vektor $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ dan $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ adalah vektor-vektor tidak nol di R^3, maka

- Penjumlahan vektor

$$\vec{a} + \vec{b} = (a_1 + b_1)\hat{\imath} + (a_2 + b_2)\hat{\jmath} + (a_3 + b_3)\hat{k}$$

- Pengurangan vektor

$$\vec{a} - \vec{b} = (a_1 - b_1)\hat{\imath} + (a_2 - b_2)\hat{\jmath} + (a_3 - b_3)\hat{k}$$

- Perkalian titik (dot product)
  a. Jika tidak diketahui sudut antara dua vektor: $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
  b. Jika diketahui sudut antara dua vektor: $\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\alpha$
- Perkalian silang (cross product)

$$\vec{a}\ x\ \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - a_3b_2)\,\hat{\imath} - (a_1b_3 - a_3b_1)\,\hat{\jmath} + (a_1b_2 - a_2b_1)\,\hat{k}$$

- Sudut antara dua vektor: $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

Latihan: *Selesaikan soal berikut*

1. Diketahui $\vec{a} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$ dan $\vec{b} = \begin{pmatrix} 1 \\ 10 \\ 2 \end{pmatrix}$. Hitunglah:
   a. $3\vec{a} + 2\vec{b}$
   b. $2\vec{a} - 3\vec{b}$
2. Diketahui $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ dan $\vec{b} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$. Hitunglah:
   a. Perkalian titik (dot product) : $\vec{a}.\vec{b}$
   b. Perkalian silang (cross product) : $\vec{a}\ x\ \vec{b}$
3. Diketahui $\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ dan $\vec{b} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$. Hitunglah besar sudut antara $\vec{a}$ dan $\vec{b}$

Jawab:

No	Penyelesaian
1a	Diketahui $\vec{a} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$ dan $\vec{b} = \begin{pmatrix} 1 \\ 10 \\ 2 \end{pmatrix}$ $3\vec{a} + 2\vec{b} = 3\begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix} + 2\begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix} + \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}$
1b	Diketahui $\vec{a} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$ dan $\vec{b} = \begin{pmatrix} 1 \\ 10 \\ 2 \end{pmatrix}$ $2\vec{a} - 3\vec{b} = 2\begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix} - 3\begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix} - \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}$
2a	$\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k} \rightarrow a_1 = \ldots\ldots\ldots\ ;\ a_2 = \ldots\ldots\ldots\ ;\ a_3 = \ldots\ldots\ldots$ $\vec{b} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k} \rightarrow b_1 = \ldots\ldots\ldots\ ;\ b_2 = \ldots\ldots\ldots\ ;\ b_3 = \ldots\ldots\ldots$ $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3 = (\ldots\ldots)(\ldots\ldots) + (\ldots\ldots)(\ldots\ldots) + (\ldots\ldots)(\ldots\ldots)$ $= \ldots\ldots\ldots + \ldots\ldots\ldots + \ldots\ldots\ldots$ $= \ldots\ldots\ldots\ldots$ Jadi, perkalian titik (dot product) = $\vec{a}.\vec{b} = \ldots\ldots\ldots\ldots$
2b	$\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k} \rightarrow a_1 = \ldots\ldots\ldots\ ;\ a_2 = \ldots\ldots\ldots\ ;\ a_3 = \ldots\ldots\ldots$ $\vec{b} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k} \rightarrow b_1 = \ldots\ldots\ldots\ ;\ b_2 = \ldots\ldots\ldots\ ;\ b_3 = \ldots\ldots\ldots$ $\vec{a}\ x\ \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - a_3b_2)\,\hat{\imath} - (a_1b_3 - a_3b_1)\,\hat{\jmath} + (a_1b_2 - a_2b_1)\,\hat{k}$ $= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{vmatrix} = ((\ldots\ldots)(\ldots\ldots) - (\ldots\ldots)(\ldots\ldots))\,\hat{\imath} - ((\ldots\ldots)(\ldots\ldots) - (\ldots\ldots)(\ldots\ldots))\,\hat{\jmath}$ $+ ((\ldots\ldots)(\ldots\ldots) - (\ldots\ldots)(\ldots\ldots))\,\hat{k}$

	$= ((......) - (......))\,\hat{\imath} - ((......) - (......))\,\hat{\jmath} + ((......) - (......))\,\hat{k}$ $= (......)\,\hat{\imath} - (......)\,\hat{\jmath} + (......)\,\hat{k}$ Perkalian silang (cross product) : $\vec{a}\ x\ \vec{b} = (......)\,\hat{\imath} - (......)\,\hat{\jmath} + (......)\,\hat{k}$
3	Diketahui $\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \rightarrow a_1 = ...,\ a_2 = ...,\ a_3 = ...$ dan $\vec{b} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} \rightarrow b_1 = ..., b_2 = ...,\ b_3 = ...$ $\lvert\vec{a}\rvert = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{(\dots\dots)^2 + (\dots\dots)^2 + (\dots\dots)^2} = \sqrt{\dots\dots + \cdots\dots + \cdots\dots} = \sqrt{\dots\dots}$ $\lvert\vec{b}\rvert = \sqrt{b_1^2 + b_2^2 + b_3^2} = \sqrt{(\dots\dots)^2 + (\dots\dots)^2 + (\dots\dots)^2} = \sqrt{\dots\dots + \cdots\dots + \cdots\dots} = \sqrt{\dots\dots}$ $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3 = (.......)(.......) + (.......)(.......) + (.......)(.......)$ $= .... + ... + ...$ $= ....$ $\cos\alpha = \frac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert} = \frac{................}{(\sqrt{........})(\sqrt{........})} = \frac{......}{\sqrt{.........}} = \frac{..........}{.........} =$............ (tiga angka di belakang koma) α = arc cos .... = ...° (satu angka di belakang koma) Jadi, besar sudut antara vektor $\vec{a}$ dan $\vec{b}$ adalah ....° (satu angka di belakang koma)