Octal and Hexadecimal Number Systems

OCTAL or **BASE-8** numbers uses eight symbols: 0, 1, 2, 3, 4, 5, 6, and 7 (count them!) and position plays a major role in expressing their meaning. For example $53{,}702_8$ means

$$\underset{\text{4096's}}{\underline{5 \times 8^4}} + \underset{\text{512's}}{\underline{3 \times 8^3}} + \underset{\text{Sixty-fours}}{\underline{7 \times 8^2}} + \underset{\text{Eights}}{\underline{0 \times 8^1}} + \underset{\text{Ones (Units)}}{\underline{2 \times 8^0}}$$

To change this number to base 10, multiply each placeholder by the amount its location represents and add: $(5 \times 4096) + (3 \times 512) + (7 \times 64) + (0 \times 8) + (2 \times 1) = 20{,}480 + 1536 + 448 + 0 + 1 = 22{,}466_{10}$

Now you try some:

436_8 = ________________(base 10)

1234_8 = ________________(base 10)

524_8 = ________________(base 10)

Base 16	A	B	C	D	E	F
Base 10	10	11	12	13	14	15

HEXADECIMAL or **BASE-16** numbers uses sixteen symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, and E (count them!) and position plays a major role in expressing their meaning. For example $537CA_{16}$ means

$$\underset{\text{65,536's}}{\underline{5 \times 16^4}} + \underset{\text{4096's}}{\underline{3 \times 16^3}} + \underset{\text{256's}}{\underline{7 \times 16^2}} + \underset{\text{Sixteens}}{\underline{C \times 16^1}} + \underset{\text{Ones (Units)}}{\underline{A \times 16^0}}$$

To change this number to base 10, multiply each placeholder by the amount its location represents and add: $(5 \times 65{,}536) + (3 \times 4096) + (7 \times 256) + (12 \times 8) + (10 \times 1) = 327{,}680 + 12{,}288 + 1792 + 96 + 10 = 341{,}866_{10}$

Now you try some:

$4B6_{16}$ = ________________(base 10)

1234_{16} = ________________(base 10)

EDA_{16} = ________________(base 10)

Changing a Decimal Number to an Octal Number

Repeatedly divide by eight and record the remainder for each division – read "answer" upwards.

Example: Rewrite the decimal number 215_{10} as an octal number.

8	215	
8	26	R=7
8	3	R=2
8	0	R=3 ↑read↑
	0	

8 divides into 215 twenty-six times with a remainder of 7; then 8 divides into 26 three times with a remainder of 2; and so forth...

The octal result is read upwards↑, therefore **$215_{10} = 327_8$**

Now you try one:

682_{10} = ___________$_8$

Changing a Decimal Number to an Hexadecimal Number

Repeatedly divide by sixteen and record the remainder for each division – read "answer" upwards.

Example: Rewrite the decimal number 215_{10} as an octal number.

16	215	
16	13	R=7
16	0	R=13_{10} = D
	0	↑read↑

16 divides into 215 thirteen times with a remainder of 7; then 16 divides into 13 zero times with a remainder of 13, which is represented in in base

The octal result is read upwards↑, therefore **$215_{10} = D7_{16}$**

Now you try one:

1682_{10} = ___________$_{16}$

Note how the above algorithms can be adapted to change a decimal number to <u>any</u> chosen base.

Changing Bases Back and Forth between Binary, Octal, and Hexadecimal Systems: An Easy Task!

1. From Binary to Octal – Count off from right to left by three and translate each triad into base 10. These digits will be the base-8 symbols to express this binary number in octal.
2. From Binary to Hexadecimal - Count off from right to left by four and translate each quad into base 10. These digits will be the base-16 symbols to express this binary number in hexadecimal.
3. From Hexadecimal OR Octal to Binary – Change each symbol to binary and you are done!
4. From Octal to Hexadecimal OR from Hexadecimal to Octal – Change the higher base to binary and then use #1 or #2 above to change the binary number to the base desired.

EXAMPLES:

a) *Change 1101001010_2 to an octal number.*

I added two zeroes on the left to create a triad.

001 101 001 010

1 5 1 2 therefore, the octal number is $\mathbf{1512_8}$

b) *Change 1001011101_2 to a hexadecimal number.*

0010 0101 1101

2 5 13/D therefore, the hexadecimal number is $\mathbf{25D_{16}}$

c) *Change $A3D9_{16}$ to a binary number.*

A 3 D 9

1010 0011 1101 1001 therefore, the binary number is $\mathbf{1010001111011001_2}$

d) *Change 630076_8 to a binary number.*

6 3 0 0 7 6

110 011 000 000 111 110 therefore, the binary number is

$\mathbf{110011000000111110_2}$

e) *Change $A45_{16}$ to octal.*

A 4 5
1010 0100 0101 (rewritten in binary)
101 001 001 101 (regrouped the binary digits into groups of three)
5 1 1 5 therefore the octal number is $\mathbf{5115_8}$

f) Change 5401_8 to hexadecimal.

5	4	0	1	
101	100	000	001	(rewritten in binary)
1011	0000	0001		(regrouped the binary digits into groups of four)
B	0	1		therefore the hexadecimal number is $\mathbf{B01_{16}}$

Further Exercises

1. Express each number as a decimal number.
   a. 263_8
   b. $B21_{16}$
   c. 5100_8
   d. $100E_{16}$
   e. 100332_8
   f. 10011_{16}

2. Express each number as a binary number.
   a. 2524_8
   b. $BAC9_{16}$
   c. 332210_8
   d. $4009D_{16}$

3. Express each number as an octal number.
   a. 101001001_2
   b. 1001010000100010_2
   c. $B78_{16}$
   d. 1234_{16}

4. Express each number as a hexadecimal number.
   a. $10101000000101010 10_2$
   b. 1010101010_2
   c. 2526_8
   d. 50004734_8