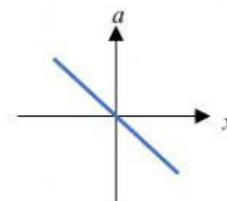


Simple Harmonic Motion

Simple harmonic motion is defined as the motion of a particle such that its **acceleration, a , is proportional to displacement, x** , from a fixed point and **is directed towards the fixed point**. In equation form, this can be written as:

$$a = -\omega^2 x$$

where ω^2 is a constant. The minus sign in the equation must be preserved and hence we take the constant as a squared quantity because this will ensure that the constant is always positive (because the square of both positive and negative number are positive). The **negative sign** shows that the **acceleration, a , is always in the opposite direction to the displacement, x** , i.e. the acceleration is **always directed towards the fixed point** from which the displacement is measured. If the equation is represented in a graph of a against x , it will be a straight line passing through origin, with negative gradient (due to the minus sign in the equation).



Newton's second law states that the force acting on a body is proportional to the acceleration of the body. Since the acceleration of the particle is proportional to its displacement, the **resultant force acting on the particle is also proportional to the displacement** and is always **acting towards the fixed point** (the same as acceleration). We call this as **restoring force** and it is experienced by object experiencing simple harmonic motion. It is also the only external force acting on a particle undergoing free oscillations.

At equilibrium position, the displacement of the pendulum is 0 and the maximum displacement of the pendulum is x_0 .

We knew from Kinematics that velocity can be found by finding the gradient of a displacement-time graph. We can see from the graph above that at maximum displacement, x_0 , the gradient is zero and when the displacement = 0, the gradient is maximum. Therefore:

- when the pendulum is at maximum displacement the velocity, $v = 0$, and
- when the pendulum is at equilibrium position the velocity is maximum, v_0 .

And then we had also learnt from Kinematics that the gradient of a velocity-time graph can be used to find the acceleration, a . At maximum speed, v_0 , the gradient is zero and when the velocity = 0, the gradient is maximum. Therefore, the pendulum has:

- largest acceleration when $v = 0$, while it is at its maximum displacement and
- $a = 0$ when the velocity is maximum, while the pendulum is at equilibrium position.