GRADE 12 EXAMINATION

QUESTION 1

1.1 Solve for $x \in R$, without the use of a calculator:

(a) $|x+3| + 2x = 4$ (7)

$x > -3 \quad x + 3 + 2x = 4$

$\therefore 3x = 1$

$\therefore x = \frac{1}{3}$

n/a

(b) $\cos^{-1}\left(\frac{x^2}{8}\right) = \frac{\pi}{3}$ (4)

$\frac{x^2}{8} = \frac{1}{2}$

(c) $lnx^2 - 3log_x e = 1$ (8)

$2lnx - \frac{3}{ln\,x} = 1$

$k = 1,5 \qquad k = -1$

$x = e^{1,5} \qquad x = e^{-1}$

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1.2 Determine the domain and range of the graph of:

$y = ln(e^2 - x^2)$ (8)

[27]

$e^2 - x^2$

$\therefore lne^2 = 2$

QUESTION 2

2.1 Simplify: $\sqrt{i^4}$ (2)

i^4

$\sqrt{1}$

2.2 Find the real values of a and b such that $(3+2i)(a+3i) = bi$ (7)

$(a + 3i)(3 + 2i) = bi$

Compare real:

$a = 2$

Compare imaginary: $2a + 9 = b$

LIVEWORKSHEETS

2.3 One of the solutions to the equation $x^2 - 2x + p = 0$ is $x = q + \sqrt{3}i$.

Find the rational values of p and q. (7)

[16]

$x = q - \sqrt{3}i$ is also a root

Sum of roots = ______

______

______ $= q^2 + 3 = p$

______

QUESTION 3

3.1 State whether each of the following statements is TRUE or FALSE:

(a) If a function is differentiable at a point, then the limit of the function must exist at that point.

(b) If a function is continuous at a point, then it must also be differentiable at that point.

(c) If the limit of the function does not exist at a point, then the graph will have an asymptote at that point.

(d) If the second derivative of a function at a point is equal to zero, then there will be a point of inflection on the graph at that point.

(e) A function can exist at a point, whether or not the limit of the function exists at that point.

(f) At a local maximum, the ***gradient*** of the graph is decreasing. (12)

3.2 A function is defined as follows:

$$f(x) = \begin{cases} p - x^2 & if \quad x \leq 2 \\ qx + 10 & if \quad x > 2 \end{cases}$$

Calculate the value(s) of p and q if f is differentiable at $x = 2$. (8)

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Continuity:

$\lim_{x \to 2^-} (p - x^2) = \lim_{x \to 2^+} (qx + 10)$

Gradients equal: $\lim_{x \to 2^-} f'(x) = \lim_{x \to 2^+} f'(x)$

$q = -4$

QUESTION 4

Alisha wants to prove by induction that $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

Her teacher has taught her the following procedure:

Step 1: Prove true for $n = 1$
Step 2: Assume true for $n = k$
Step 3: Prove true for $n = k + 1$
Step 4: Conclude the proof.

Show Alisha's working for Step 3.

Prove true for n = k + 1

$\frac{1}{\square} + \frac{1}{\square} + \frac{1}{\square} + \cdots + \frac{1}{k(k+1)} + \frac{1}{\square} = \frac{k}{\square} + \frac{1}{\square}$

$\square = \frac{k(k+2)+1}{\square} = \frac{(k+1)^2}{\square} = \frac{k+1}{k+2} = \square$

LIVEWORKSHEETS

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