GRADE 12 EXAMINATION

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 Given the matrix equation: $\begin{pmatrix} 1 & -1 \\ 2 & 4 \\ x & y \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 10 & z \\ 4 & -11 \end{pmatrix}$

Calculate the values of x, y and z.

$2 \times 4 + 4 \times (-1) = z$

$x + 2y = 4$ and $4x - y = -11$

For each of the following matrix identities, assume the operations are possible. State if the following identities are necessarily TRUE or possibly FALSE:

(a) $AB = BA$ (2)

(b) $AB = AC$ implies that $B = C$ (2)

(c) $A^{-1} \cdot A = A \cdot A^{-1} = I$ (2)

(d) $A(BC) = (AB)C$ (2)

[16]

QUESTION 2

2.1 Consider the transformation represented by the matrix equation:

$$\begin{pmatrix} 1 & -1 & 2 \\ 4 & 3 & 0 \end{pmatrix} + \begin{pmatrix} +3 & +3 & +3 \\ -1 & -1 & -1 \end{pmatrix} = M$$

(a) Describe this transformation in words. (3)

(b) Is this a rigid transformation? (1)

(c) Write down matrix M, the image of the figure after the transformation. (2)

$$\begin{pmatrix} 4 & 2 & 5 \\ 3 & 2 & -1 \end{pmatrix}$$

2.2 Consider the transformation represented by the matrix equation:

$$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 2 \\ 4 & 3 & 0 \end{pmatrix} = N$$

(a) Describe this transformation in words. (3)

(b) Is this a rigid transformation? (1)

(c) Which number related to a transformation matrix gives the scale factor by which the area of the figure must be multiplied in order to determine the area of its image? (1)

(d) Calculate the number mentioned in Question 2.2 (c). Explain the numerical relationship between the area of this figure and the area of its image. (3)

2.3 A figure in a Cartesian plane is to be rotated 30° anti-clockwise about the origin, followed by a reflection in the line $y = 3x$. Determine a single matrix that would produce these transformations in the stated order. Give the elements of this matrix correct to two decimal places. (8)

[22]

$$\begin{pmatrix} \cos 143{,}13 & \sin 143{,}13 \\ \sin 143{,}13 & -\cos 143{,}13 \end{pmatrix}\begin{pmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{pmatrix} = \begin{pmatrix} -0{,}39 & 0{,}92 \\ 0{,}92 & 0{,}39 \end{pmatrix}$$

QUESTION 3

Karyn needs to solve three linear equations simultaneously for the variables x, y and z. In order to do this, she designs the following matrix equation:

$$\begin{pmatrix} 6 & 2 & -3 \\ 6 & 3 & 1 \\ 9 & 3 & t \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 15 \\ w \end{pmatrix}$$

3.1 Karyn realises that for $t = -2$ and $w = 8$ the matrix equation has a unique solution. Determine this solution through calculation. Do **not** merely state the solution; show at least one line of working using the given matrix equation. (8)

$$\begin{pmatrix} -0,6 & -0,333 & 0,733 \\ -0,6 & 0 & 0,4 \end{pmatrix}$$

3.2 Calculate the value of t for which the matrix equation does **not** have a unique solution, irrespective of the value of w. (6)

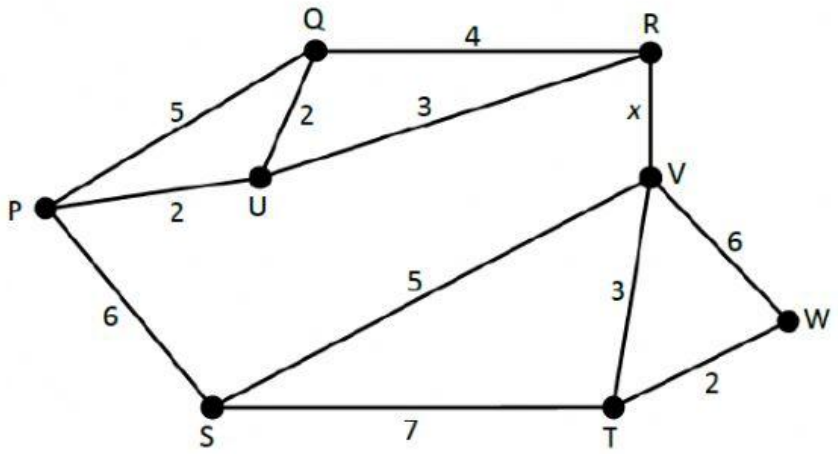
$$6(3t-3) - 2(6t-9) - 3(18-27) = 0$$
$$6t + 27 = 0$$
$$t = -4,5$$

3.3 Upon further investigation, Karyn realises that using the value of t obtained in Question 3.2, the matrix equation could in fact have an infinite number of solutions. State the corresponding value of w for this to be true. (2)

[16]

QUESTION 4

In the graph below, the path of least weight between vertex P and vertex W is to be determined. The weight of edge RV is unknown and represented by x.



4.1 In the circuit PUQ, edge PQ can be ignored as PQ > PU + QU. What other edge in the graph can be ignored for a similar reason? (2)

4.2 Why can the weights of the edges **not** represent linear distances? (2)

4.3 Design a Hamiltonian circuit on this graph, starting at W. (4)

4.4 Calculate the maximum integer weight of RV so that P → U → R → V → T → W becomes the path of least weight. (6)

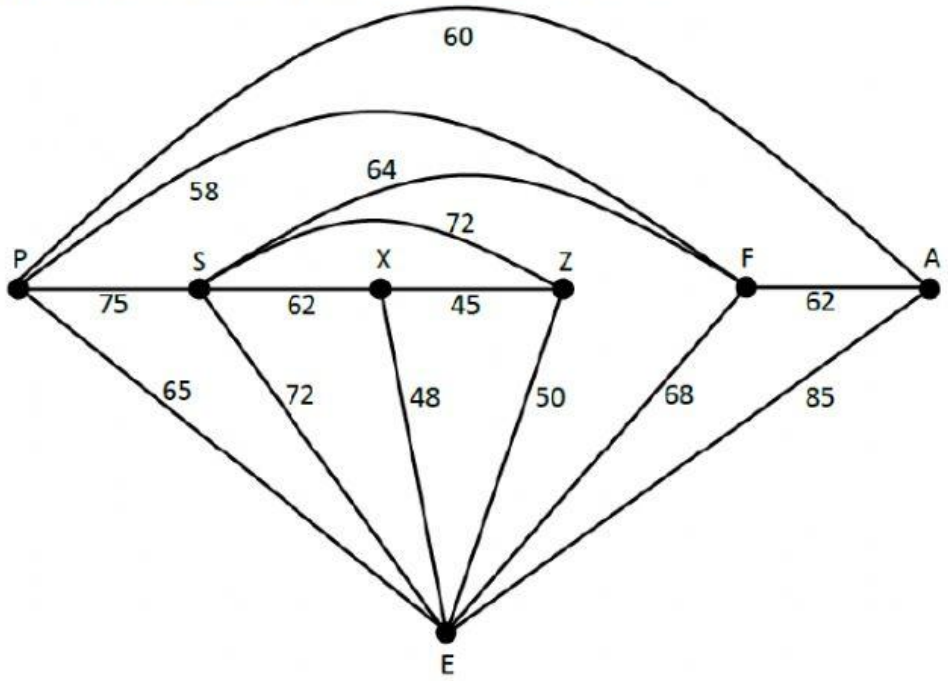
[14]

QUESTION 5

At African Union summits, five official languages are used: Arabic (A), English (E), French (F), Portuguese (P) and Swahili (S). South African delegates also use isiXhosa (X) and isiZulu (Z).

Documents need to be translated from their original language into each of the other six languages, and then back into the original language to check for inaccuracies.

In the graph below, languages are represented by the vertices. Each edge represents a direct translation between the two languages joined by the edge. The weight of the edges represents the average time taken in minutes for translation.



5.1 Which two of the seven languages are the most versatile for translation? (2)

5.2 Starting at English, determine an upper bound for the time needed to translate a document into all other languages. Use the Nearest Neighbour Algorithm and clearly record the order in which edges are selected. (8)

E → X (48) → Z (45) → S (72) → F (64) → P (58) → A (60) → E (85)

5.3 Starting at English, use inspection to determine a 'good route' for the time needed for translation. Your solution should be at least 20 minutes quicker than the upper bound calculated in Question 5.2. (10)

[20]

E → P (65) → A (60) → F (62) → S (64) → X (62) → Z (45) → E (50)