GRADE 12 EXAMINATION

MODULE 2 STATISTICS

QUESTION 1

1.1 In a large city one person in five is left-handed.

(a) Calculate the probability that in a random sample of 10 people, exactly three will be left-handed. (5)

1.1 (a) $P(X=3)=\binom{10}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^7$

(b) Find the most likely number of left-handed people in a random sample of 15 people. (2)

$15\left(\frac{1}{5}\right)$

c) In a random sample of n people, the probability that the sample contains at least one left-handed person is greater than 0,95. Solve for n, and hence determine the least number of people required in the sample. (8)

$P(x \geq 1) >$

$1-\binom{n}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^n >$

$\left(\frac{4}{5}\right)^n < 0,05$

$n\log\frac{4}{5} < \log 0,05$

$n > 13,4251$

1.2 Bjorn saves his digital images on his computer in three separate folders, namely 'Family', 'Friends' and 'Rowing'. His Family folder contains three images, his Friends folder contains four images and his Rowing folder contains eight images. All the images are different.

(a) How many ways can he arrange these 15 images in a row across his computer screen if he keeps the images from each folder together? (5)

(b) Calculate the probability that if Bjorn chooses six images at random, there are two from each folder. (5)

$$\frac{\binom{3}{2}\binom{4}{2}\binom{8}{2}}{\binom{15}{6}} = \frac{72}{715} \text{ or}$$

(c) Calculate the number of different ways in which Bjorn can choose six of these images if there are at least three images from the Rowing folder and at least one image from each of the other two folders. (8)

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$$\binom{8}{3}\binom{3}{1}\binom{4}{2} + \binom{8}{3}\binom{3}{2}\binom{4}{1} + \binom{8}{4}\binom{3}{1}\binom{4}{1}$$

QUESTION 2

2.1 It is known that the wind causes a 'chill factor' so that the human body experiences the temperature to be lower than the actual temperature. The following table gives the experienced temperature (t °C) for different wind speeds (w km/h) when the actual temperature is 10 °C. **(Work accurately to two decimal places in this question.)**

w (km/h)	0	5	10	15	20	25
t (°C)	10°	6°	–6°	–16°	–22°	–25°

(a) Determine the equation of a suitable regression line from which a value of t can be estimated for a given value w. (4)

(b) Find the value of the correlation coefficient for the data, and state what this indicates about the data. (2)

(c) Estimate the experienced temperature when the wind speed is 35 km/h. (2)

$y = 10,38 - 1,54(35)$

(d) Comment on the reliability of this estimate.

2.2 Every day Timello tries to phone his friend Nicky. Every time he phones there is a 50% chance that Nicky will answer. If Nicky answers, Timello does not phone again on that day. If Nicky does not answer, Timello tries to phone again in a few minutes' time. If Nicky has not answered after four attempts, Timello does not try again on that day.

(b) Let X be the number of unanswered phone calls made by Timello on a given day. Copy and complete the table, showing the probability distribution of X.

x	0	1	2	3	4
$P(X = x)$		$\frac{1}{4}$			

(4)

(c) Calculate the expected number of unanswered phone calls on a day if the formula for the Expected Value of the random variable X is given as:

$E[X] = \sum P(X = x) \cdot x$ (2)

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$$E[X] = 0\left(\frac{1}{2}\right) + 1\left(\frac{1}{4}\right) + 2\left(\frac{1}{8}\right) + 3\left(\frac{1}{16}\right) + 4\left(\frac{1}{16}\right)$$

QUESTION 3

3.1 In a survey of cars parked at a hockey match, 200 cars out of a random sample of 250 cars have been fitted with a tracking device.

(a) Calculate a 96% confidence interval for the population proportion of parked cars fitted with a tracking device. (6)

A 96% confidence interval for p is:

$$0,8 \pm 2,05\sqrt{\frac{(0,8)(0,2)}{250}}$$

(b) Describe, in words, what this confidence interval means statistically.

3.2 A random variable X is normally distributed with a mean μ and a standard deviation σ.

(a) Given that $5\sigma = 3\mu$, find $P(X < 2\mu)$. (8)

$$\sim N(\mu ; \sigma^2)$$

$$\sigma = \frac{3}{5}\mu$$

$$P(x < 2\mu) = P\left(z < \frac{2\mu - \mu}{\frac{3}{5}\mu} \right)$$

$$= P(z < 1,67)$$

(b) Given that the standard deviation, σ, is 2 and that $P\left(X > \frac{1}{3}\mu\right) = 0,8$.

Calculate the mean, μ.

$$-0,84 = \frac{\frac{1}{3}\mu - \mu}{2}$$

$$\frac{-42}{52} = -0,84 \times 2 = \frac{-2}{3}\mu$$

QUESTION 4

Last year Gareth found that his journey to work took on average 45,7 minutes with a standard deviation of 3,2 minutes. Gareth wishes to test whether his travel time has increased this year. He notes the time, in minutes, for a random sample of eight journeys this year, with the following results.

46,2 41,7 49,2 47,1 47,2 48,4 53,7 45,5

It may be assumed that the population of this year's journey times is normally distributed also with a standard deviation of 3,2 minutes.

4.1 State, with a reason, whether Gareth should use a one-tailed or a two-tailed hypothesis test. (2)

4.2 Determine whether the sample provides significant evidence, at the 4% level of significance, that Gareth's travel time has increased this year. Specifically mention the conclusion of Gareth's findings. (10)

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$H_0 : \mu = 45,7$

$H_1 : \mu > 45,7$

Rejection region: reject H_0 if $z > 1,75$

Test Statistic:

$$z = \frac{47,4 - 45,7}{\frac{3,2}{\sqrt{8}}} = 1,5026$$