GRADE 12 EXAMINATION

ADVANCED PROGRAMME MATHEMATICS: PAPER II

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 Given $\begin{pmatrix} 2 & 8 & 3 \\ 5 & 4 & 1 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ y & 3 \\ x & 4 \end{pmatrix} = \begin{pmatrix} 38 & 38 \\ 46 & 21 \end{pmatrix}$, solve for x and y. (7)

1.2 To solve the following equations simultaneously,

$$x + 2y - z = -1$$
$$2x + 6y + z = 7$$
$$5x + 7y - 4z = 9$$

we apply Gaussian row-reduction to the augmented matrix, $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 2 & 6 & 1 & 7 \\ 5 & 7 & -4 & 9 \end{array}\right)$

Step 1: $R_2 - 2R_1$ $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 2 & 3 & 9 \\ 5 & 7 & -4 & 9 \end{array}\right)$

Step 2: $R_3 - 5R_1$ $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 2 & 3 & 9 \\ 0 & -3 & 1 & 14 \end{array}\right)$

Step 3: $R_1 - R_2$ $\left(\begin{array}{ccc|c} 1 & 0 & -4 & -10 \\ 0 & 2 & 3 & 9 \\ 0 & -3 & 1 & 14 \end{array}\right)$

Continue the **Gaussian row reduction**, to solve for the variables x, y and z. (8)

QUESTION 2

2.1 Given, $P = \begin{bmatrix} 2 & 2 & 6 \\ 4 & 2 & 2 \end{bmatrix}$, calculate P', the image of P, using only matrix algebra,

(a) Translated one unit left and three units down.

(b) Reflected in the $y = -x$.

2.2 Find a simplified single matrix that first enlarges a shape by a factor of $\frac{2}{\sqrt{2}}$ and then rotates it 225°. (8)

2.3 A line with end points (3; 4) and (–1; 0) is reflected to points $\left(\frac{-3+4\sqrt{3}}{2};\frac{4+3\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2};\frac{-\sqrt{3}}{2}\right)$ respectively. Find the gradient of the line of reflection. (12)
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QUESTION 3

3.1 State whether the given statements are true or false, giving a reason if false.

(a) In a given square matrix, if you multiply a row or column by a non-zero constant, the determinant is multiplied by that same non-zero constant. (2)

(b) If you interchange two columns in a determinant, the resulting determinant will be the same. (2)

(c) The determinant of a matrix will be zero if a row is a constant multiple of another row. (2)

3.2 Given $A = \begin{pmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 \\ 9 & 2 & 3 & 1 \end{pmatrix}$

Joe produced the following first step in calculating the determinant of a 4×4 matrix.

$$\begin{vmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 \\ 9 & 2 & 3 & 1 \end{vmatrix} = p\begin{vmatrix} 4 & 1 & 2 \\ 3 & 2 & 1 \\ 9 & 3 & 1 \end{vmatrix} + q\begin{vmatrix} 3 & 0 & 1 \\ 4 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix}$$

(a) Joe applied his knowledge of determinants to considerably simplify the process of finding the determinant. Describe what property of determinants Joe applied to produce his first step. (2)

(b) Determine the value of p and q.

(c) Hence, using matrix algebra, show **that the determinant is 24.**

QUESTION 4

Select the most correct answer in each question:

4.1 What is the number of edges present in a complete graph with n vertices?

(a) $\frac{n(n+1)}{2}$

(b) $\frac{n(n-1)}{2}$

(c) n

(d) The information given is insufficient