

LKPD 01

"RUMUS JUMLAH DAN SELISIH DUA SUDUT"

NAMA LENGKAP

KELAS

NO. ABSEN

KOMPETENSI DASAR

3.2. Membedakan penggunaan jumlah dan selisih sinus dan cosinus.

4.2. Menyelesaikan masalah yang berkaitan dengan rumus jumlah dan selisih sinus dan cosinus.

INDIKATOR

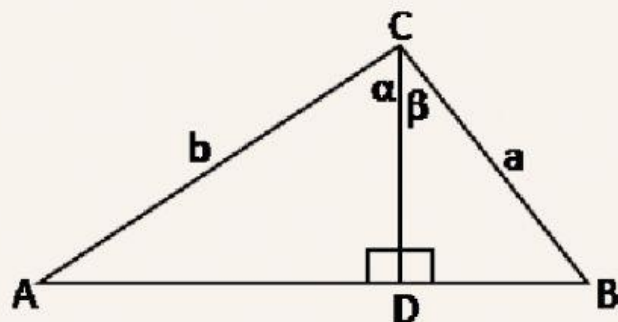
3.2.1 Menemukan rumus jumlah dan selisih dua sudut.

TUJUAN PEMBELAJARAN

Diharapkan agar peserta didik dapat menemukan rumus jumlah dan selisih dua sudut serta memiliki sikap berakhlak mulia, bernalar kritis dan dapat memecahkan masalah secara mandiri dan kreatif.

A. RUMUS $\sin(\alpha \pm \beta)$

Perhatikan segitiga ABC di bawah ini



➤ Nyatakan CD, AD, dan BD dalam perbandingan trigonometri

$$\cos \alpha = \frac{\boxed{}}{AC}$$

$$\cos \alpha = \frac{\boxed{}}{\boxed{}}$$

$$\boxed{} = \boxed{} \times \cos \alpha$$

$$\cos \beta = \frac{CD}{\boxed{}}$$

$$\cos \beta = \frac{\boxed{}}{\boxed{}}$$

$$\boxed{} = \boxed{} \times \cos \beta$$

$$\sin \alpha = \frac{\boxed{}}{AC}$$

$$\sin \alpha = \frac{\boxed{}}{\boxed{}}$$

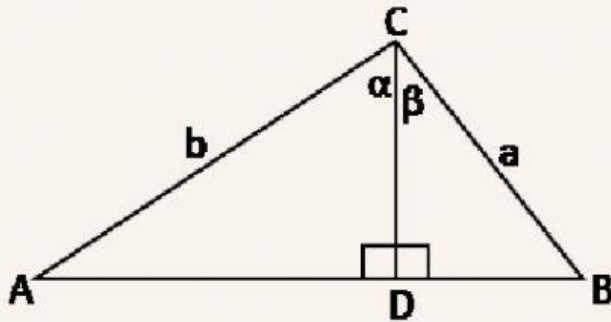
$$\boxed{} = \boxed{} \times \sin \alpha$$

$$\sin \beta = \frac{BD}{\boxed{}}$$

$$\sin \beta = \frac{\boxed{}}{\boxed{}}$$

$$\boxed{} = \boxed{} \times \sin \beta$$

➤ Nyatakan luas $\triangle ADC$, luas $\triangle BDC$, dan luas $\triangle ABC$



Luas $\triangle ADC$

$$= \frac{1}{2} \times \square \times \square$$

$$= \frac{1}{2} \times \square \times \square \alpha \times \square \times \square \beta$$

$$= \frac{1}{2} \times a \times \square \times \square \alpha \times \square \beta$$

Luas $\triangle BDC$

$$= \frac{1}{2} \times \square \times \square$$

$$= \frac{1}{2} \times \square \times \square \alpha \times \square \times \square \beta$$

$$= \frac{1}{2} \times a \times \square \times \square \beta \times \square \alpha$$

Luas $\triangle ABC$

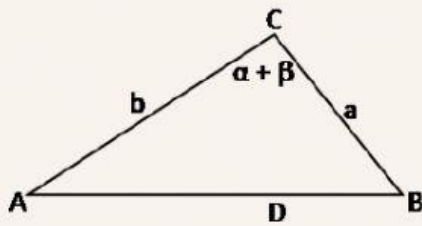
$= \text{Luas } \triangle ADC + \text{Luas } \triangle BDC$

$$= \left(\frac{1}{2} \times \square \times \square \times \square \alpha \times \square \beta \right) + \left(\frac{1}{2} \times \square \times \square \times \square \beta \times \square \alpha \right)$$

$$= \frac{1}{2} ab (\square \alpha \times \square \beta) + \frac{1}{2} ab (\square \beta \times \square \alpha)$$

$$= \frac{1}{2} ab ((\square \alpha \times \square \beta) + (\square \beta \times \square \alpha))$$

➤ Nyatakan luas $\triangle ABC$ menggunakan rumus aturan sinus

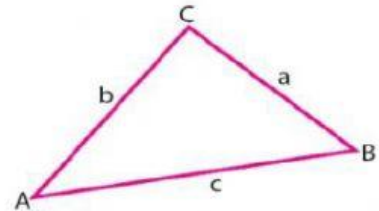


Luas Segitiga Sembarang

$$L_{\triangle ABC} = \frac{1}{2}bc \cdot \sin A$$

$$L_{\triangle ABC} = \frac{1}{2}ac \cdot \sin B$$

$$L_{\triangle ABC} = \frac{1}{2}ab \cdot \sin C$$



$$\text{Luas } \triangle ABC = \frac{1}{2} \times \square \times \square \times \sin C$$

$$\text{Luas } \triangle ABC = \frac{1}{2} \times \square \times \square \times \square (\alpha + \beta)$$

$$\frac{1}{2}ab \left((\square \alpha \times \square \beta) + (\square \beta \times \square \alpha) \right) = \frac{1}{2}ab \times \square (\alpha + \beta)$$

$$(\square \alpha \times \square \beta) + (\square \beta \times \square \alpha) = \square (\alpha + \beta)$$

➤ Jika β diganti dengan $(-\beta)$, maka :

$$\sin(\alpha + (-\beta)) = \square \alpha \times \square (-\beta) + \square (-\beta) \times \square \alpha$$

$$\sin(\alpha - \beta) = \square \alpha \times \square \beta - \square \beta \times \square \alpha$$

B. RUMUS $\cos(\alpha \pm \beta)$

Dengan menggunakan rumus sudut berelasi (pelajaran Trigonometri di kelas X), kita dapat menemukan rumus cosinus jumlah dua sudut sebagai berikut.

$$\cos \alpha = \sin(90^\circ - \alpha), \text{ sehingga}$$

$$\cos(\alpha + \beta) = \sin(\text{ } - (\alpha + \beta))$$

$$= \sin((\text{ } - \alpha) - \beta)$$

$$= \sin(\text{ } - \alpha) \cos \beta - \cos(\text{ } - \alpha) \sin \beta$$

$$= \text{ } \alpha \cos \beta - \text{ } \alpha \sin \beta$$

$$\text{Jadi, } \cos(\alpha + \beta) = \text{ } \alpha \cos \beta - \text{ } \alpha \sin \beta$$

➤ Jika β diganti dengan $(-\beta)$, maka :

$$\cos(\alpha + (-\beta)) = \text{ } \alpha \times \cos(-\beta) - \text{ } \alpha \times \sin(-\beta)$$

$$\cos(\alpha - \beta) = \text{ } \alpha \times \cos \beta + \text{ } \alpha \times \sin \beta$$

C. RUMUS $\tan(\alpha \pm \beta)$

Dengan menggunakan rumus sinus dan kosinus untuk jumlah dan selisih dua sudut, maka :

$$\tan(\alpha + \beta) = \frac{\text{ } (\alpha + \beta)}{\text{ } (\alpha + \beta)}$$

$$= \frac{\text{ } \alpha \text{ } \beta + \text{ } \beta \text{ } \alpha}{\text{ } \alpha \text{ } \beta - \text{ } \alpha \text{ } \beta}$$

$$= \frac{\text{ } \alpha \text{ } \beta + \text{ } \beta \text{ } \alpha}{\text{ } \alpha \text{ } \beta - \text{ } \alpha \text{ } \beta} \times \frac{\frac{1}{\cos \alpha \cos \beta}}{\frac{1}{\cos \alpha \cos \beta}}$$

$$\begin{aligned}
 & \frac{\boxed{}\alpha\boxed{}\beta + \boxed{}\beta\boxed{}\alpha}{\cos\alpha\cos\beta} \\
 = & \frac{\boxed{}\alpha\boxed{}\beta - \boxed{}\alpha\boxed{}\beta}{\cos\alpha\cos\beta} \\
 = & \frac{\boxed{}\alpha + \boxed{}\beta}{1 - \boxed{}\alpha\boxed{}\beta}
 \end{aligned}$$

$$\begin{aligned}
 \tan(\alpha - \beta) &= \frac{\boxed{}(\alpha - \beta)}{\boxed{}(\alpha - \beta)} \\
 &= \frac{\boxed{}\alpha\boxed{}\beta - \boxed{}\beta\boxed{}\alpha}{\boxed{}\alpha\boxed{}\beta + \boxed{}\alpha\boxed{}\beta} \\
 &= \frac{\boxed{}\alpha\boxed{}\beta - \boxed{}\beta\boxed{}\alpha}{\boxed{}\alpha\boxed{}\beta + \boxed{}\alpha\boxed{}\beta} \times \frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}} \\
 &= \frac{\boxed{}\alpha\boxed{}\beta - \boxed{}\beta\boxed{}\alpha}{\cos\alpha\cos\beta} \times \frac{1}{\cos\alpha\cos\beta} \\
 &= \frac{\boxed{}\alpha\boxed{}\beta - \boxed{}\beta\boxed{}\alpha}{\boxed{}\alpha\boxed{}\beta + \boxed{}\alpha\boxed{}\beta} \\
 &= \frac{\boxed{}\alpha - \boxed{}\beta}{1 + \boxed{}\alpha\boxed{}\beta}
 \end{aligned}$$